Please answer each question in as much detail as necessary to provide a complete answer, but without gratuitous padding. You will have no more than 1 hour 40 minutes to finish the exam. Good luck.

1. In class, we derived relationships among housing production, land rents and housing prices such that:

$$\frac{\partial \ln (K(u)/L(u))}{\partial u} = \sigma \frac{\partial \ln R(u)}{\partial u} = \frac{\sigma}{\rho_L} \frac{\partial \ln p(u)}{\partial u}$$

where $\sigma$ is the elasticity of substitution and $\rho_L$ is the land share of the housing price. $u$ is distance from the central place.

Suppose that housing is constructed with the production function:

$$Q = K^{0.8}L^{0.2}$$

a. Calculate the elasticity of substitution of capital for land.

b. What is the relationship between land rent and the capital-labor ratio?

c. What is the relationship between housing price and the capital-labor ratio? Why?
2. Consider land rents in an urban area such that:

\[ R_0 \]

\[ R(u) \]

\[ R_{ai} \]

\[ 0 \quad \bar{u} \]

\[ u = distance \]

Use the above diagram to aid your analyses.

a. What are the primary distinctions between the “closed city” and the “open city” in the analyses?

b. Suppose that incomes rise in an open city. What will happen to the utility levels of the population? What will happen to land rents at various locations, and to aggregate (the sum of all of the) land rents in the city?

c. Suppose that incomes rise in a closed city (i.e. no immigration). What will happen to the utility levels of the population? What will happen to land rents at various locations, and to aggregate (the sum of all of the) land rents compared with the open city.
3. DiPasquale and Wheaton develop a 4-quadrant development model that relates the property and the asset markets.

a. Derive the model carefully, noting how only one point in Quadrant I is consistent with equilibrium in all of the markets.

b. Suppose that economic growth leads to an increased demand for rental property. Trace the impact throughout the model.

c. Suppose that interest rates fall. Trace the impact throughout the model.
4. Economists have made considerable use of density functions of the form:

\[ D(u) = D_0 e^{-\gamma u} \]

a. Discuss at least two strengths and at least two weaknesses of the use of density functions for urban analysis.

b. A city of 50,000 has a value of \( \gamma \) equal to 1, and a city of 100,000 has a value of \( \gamma \) equal to 0.5. Which one is more centralized? Why?

c. The city limits of the city with 100,000 people (with \( \gamma = 0.5 \)) are at \( \bar{u} = 4 \). Calculate \( D_0 \).

5. Consider the article that you presented in class.

a. Discuss briefly one or more strengths of the article that you presented.

b. Discuss briefly one or more weaknesses of the article that you presented.

c. If you were extending the research that was presented, how would you improve on the analyses and the results.