Concave-Monotone Treatment Response and Monotone Treatment Selection: With Returns to Schooling Application*

TSUNAO OKUMURA†

International Graduate School of Social Sciences, Yokohama National University, Japan

EMIKO USUI‡

Department of Economics, Wayne State University, Detroit Michigan USA

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Abstract

This paper identifies the sharp bounds on the mean treatment response under concave monotone treatment response (concave-MTR) and monotone treatment selection (MTS) assumptions. Empirical application to the mean returns to schooling shows that the estimates of our bounds are substantially narrower than (1) the estimates using only the concave-MTR assumption of Manski (1997) and (2) the estimates using only MTR and MTS assumptions of Manski and Pepper (2000). Our estimates are close to the point estimates from the previous empirical studies.

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†Corresponding author: Tsunao Okumura, International Graduate School of Social Sciences, Yokohama National University, 79-4 Tokiwadai Hodogayaku, Yokohama 2408501, Japan. Phone and Fax: 81-45-339-3524. Email: okumura@ynu.ac.jp.

‡Emiko Usui, Department of Economics, Wayne State University, Detroit MI 48202 USA. Phone: 1-313-577-3349. Fax: 1-313-577-9564. Email: usui@wayne.edu.

1 Introduction

This paper identifies the sharp bounds on the mean treatment response under the concave monotone treatment response (concave-MTR) and monotone treatment selection (MTS) assumptions. Empirical application to the mean returns to schooling shows that the estimates of our bounds are substantially narrower than the estimates using only the concave-MTR assumption of Manski (1997), and the estimates using only MTR and MTS assumptions of Manski and Pepper (2000). We, therefore, study the extent to which introducing the assumption of concave treatment functions into the assumptions of MTR and MTS narrows Manski and Pepper's (2000) bounds from both a theoretical and an empirical viewpoint. It is commonly assumed in economic analysis that the function is concave in input components, since it satisfies the diminishing marginal returns and leads to a unique optimal solution.

Manski (1997) studied the sharp bounds on the distributions and parameters respecting stochastic dominance including the means of treatment response. These response functions are assumed to be monotone, semi-monotone and concave-monotone in treatment. Manski and Pepper (2000) introduced the assumption of monotone instrumental variables (MIV). In order to investigate the sharp bounds on the mean treatment response, they combined the assumptions of monotone treatment selection (MTS), which is a special case of MIV and monotone treatment response (MTR). They applied their method to an estimation of the returns to schooling. Manski and Pepper's (2000) bound estimates are narrower than those of Manski (1997), yet large enough that "the MTS-MTR assumption does not, ..., have sufficient identifying power" (Manski and Pepper [p.1009, 2000]) by comparison with the point estimates of the returns to schooling in the existing empirical literature.

Using the 2000 wave of the National Longitudinal Survey of Youth (NLSY), we implement our econometric model to estimate the returns to schooling. Subsequently, we compare the estimates of our bounds with those of Manski (1997) and Manski and Pepper (2000). Our estimates are much narrower and closer to the point estimates of previous empirical studies.

In Section 2 we study the sharp bounds on the mean treatment functions under the assumptions of concave monotone treatment response and monotone treatment selection. Section 3 applies the approach to the estimation of the returns to schooling. We conclude with Section 4.

2 Concave-Monotonicity and Monotone Treatment Selection

The same setup as Manski (1997) and Manski and Pepper (2000) is employed. There is a probability space (J, Ω, P) of individuals. Each member j of population J has an individual-specific response function $y_j(\cdot) : T \to Y$ mapping the mutually exclusive and exhaustive treatments $t \in T$ into outcomes $y_j(t) \in Y$. Individual j has a realized treatment $z_j \in T$ and a realized outcome $y_j \equiv y_j(z_j)$, both of which are observable. The latent outcomes $y_j(t)$, $t \neq z_j$ are not observable. By observing (z, y) for a random sample of individuals, an analyst can empirically learn about distribution $P(z, y)$. By combining this empirical evidence with prior information, one can inquire information about the distribution $P[y(\cdot)]$ of response functions and specifically the mean treatment response $E[y(\cdot)]$.

Manski (1997) assumed monotone treatment response (MTR):
Let T be an ordered set and t_1 and t_2 be elements of T. For each $j \in J$,

$$t_2 \geq t_1 \Longrightarrow y_j(t_2) \geq y_j(t_1).$$

Under MTR assumption, he showed the sharp bounds on $E[y(t)]$:

$$\begin{aligned} & \sum_{s \leq t} E[y| z = s] P(z = s) + y_0 P(z > t) \\ \leq \quad & E[y(t)] \leq \sum_{s \geq t} E[y| z = s] P(z = s) + y_1 P(z < t), \end{aligned}$$

where $[y_0, y_1]$ is the range of Y.

Manski (1997) also showed the sharp bounds on $E[y(t)]$ when $y_j(\cdot)$ is a concave and monotone treatment response (concave-MTR), $T = [0, \lambda]$ for some $\lambda \in (0, \infty]$ and $Y = [0, \infty]$:

$$\begin{aligned} & \sum_{s < t} E[y| z = s] P(z = s) + E\left[\left.\frac{y}{z} t \right| z \geq t\right] P(z \geq t) \\ \leq \quad & E[y(t)] \leq \sum_{s \geq t} E[y| z = s] P(z = s) + E\left[\left.\frac{y}{z} t \right| z < t\right] P(z < t). \end{aligned}$$

Manski and Pepper (2000) introduced the assumption of monotone treatment selection (MTS):

$$t_2 \geq t_1 \Longrightarrow E[y(t)| z = t_2] \geq E[y(t)| z = t_1].$$

Under MTS and MTR assumptions, they showed the sharp bounds on $E[y(t)]$:

$$\begin{aligned} & \sum_{s<t} E[y|\, z=s]\, P(z=s) + E[y|\, z=t]\, P(z \geq t) \\ \leq \quad & E[y(t)] \leq \sum_{s>t} E[y|\, z=s]\, P(z=s) + E[y|\, z=t]\, P(z \leq t). \end{aligned}$$

In contrast, we assume the concave-MTR and MTS. The following proposition shows the sharp bounds on $E[y(t)]$ under the concave-MTR and MTS assumptions. The basic idea is the following: The MTS assumption implies that, for $s_1 < s_2$, $E[y|\, z=s_1] \leq E[y(s_1)|\, z=s_2]$. The concave-MTR assumption implies $E[y(\tau)|\, z=s_2]$ is concave-MTR in $\tau \in T$. Thus, the straight line traversing $(s_1, E[y|\, z=s_1])$ and $(s_2, E[y|\, z=s_2])$ is the lower bound on $E[y(t)|\, z=s_2]$ for $s_1 \leq t \leq s_2$ and the upper bound for $t \geq s_2$.

Proposition 1 *Let T be ordered. Let $T=[0,\lambda]$ for some $\lambda \in (0,\infty]$ and $Y=[0,\infty]$. Assume that $y_j()$, $j \in J$, satisfies the concave-monotone treatment and MTS assumptions. Then, for $(t,s,s',u) \in T \times T \times T \times T$,*

$$\begin{aligned} & \sum_{s<t} E[y|\, z=s]\, P(z=s) \\ & + \sum_{s \geq t} \max_{\{s' | s \geq s' \geq t\}} \left(E[y|\, z=s'] + \max_{\{u|u<t\}} \left\{ \frac{E[y|\, z=s'] - E[y|\, z=u]}{s'-u} (t-s') \right\} \right) P(z=s) \\ \leq \quad & E[y(t)] \\ \leq \quad & \sum_{s>t} E[y|\, z=s]\, P(z=s) \\ & + \sum_{s \leq t} \min_{\{s' | s \leq s' \leq t\}} \left\{ E[y|\, z=s'] + \min_{\{u|u<s'\}} \frac{E[y|\, z=s'] - E[y|\, z=u]}{s'-u} (t-s') \right\} P(z=s), \end{aligned}$$

where we define $E[y|\, z=0]=0$.

These bounds are sharp. Furthermore, these bounds are narrower than or equal to those using only the concave-MTR assumption of Manski (1997), as well as those using only MTR and MTS assumptions of Manski and Pepper (2000).

Proof.

For $u < s$, $E[y(u)|\, z = s] \geq E[y(u)|\, z = u] = E[y|\, z = u]$ by MTS.

$E[y|\, z = s] = E[y(s)|\, z = s] \geq E[y(u)|\, z = s]$ by MTR.

Hence, $E[y|\, z = s] \geq E[y(u)|\, z = s] \geq E[y|\, z = u]$.

Since $y_j(t)$ is concave-monotone for all $j \in J$, $E[y(t)|\, z = s]$ is concave-monotone in t.

Therefore, for $t \geq s > u$,

$$E[y(t)|\, z = s] \leq E[y|\, z = s] + \frac{E[y|\, z = s] - E[y|\, z = u]}{s - u}(t - s), \tag{1}$$

and for $s \geq t > u$,

$$E[y(t)|\, z = s] \geq E[y|\, z = s] + \frac{E[y|\, z = s] - E[y|\, z = u]}{s - u}(t - s). \tag{2}$$

Since Equation (1) holds for any $u < s \leq t$, for $t \geq s$,

$$E[y(t)|\, z = s] \leq E[y|\, z = s] + \min_{\{u|u<s\}} \frac{E[y|\, z = s] - E[y|\, z = u]}{s - u}(t - s). \tag{3}$$

Similarly, since Equation (2) holds for any $u < t \leq s$, for $t \leq s$,

$$E[y(t)|\, z = s] \geq E[y|\, z = s] + \max_{\{u|u<t\}} \left\{ \frac{E[y|\, z = s] - E[y|\, z = u]}{s - u}(t - s) \right\}. \tag{4}$$

As Manski (1997) showed, because $T = [0, \lambda]$ and $Y = [0, \infty]$, for $t \geq s$,

$$E[y(t)|\, z = s] \leq E[y|\, z = s] + \frac{E[y|\, z = s]}{s}(t - s) = E\left[\left. \frac{y}{z}t \right| z = s \right],$$

and for $t \leq s$,

$$E[y(t)|\, z = s] \geq E[y|\, z = s] + \frac{E[y|\, z = s]}{s}(t - s) = E\left[\left. \frac{y}{z}t \right| z = s \right].$$

By defining $E[y|\, z = 0] = 0$, these bounds have been included in Equations (3) and (4).

MTS implies that the upper and lower bounds on $E[y(t)|\, z = s]$ are weakly increasing in s.

Thus, Equations (3) and (4) imply that, for $t \geq s$,

$$E[y(t)|\, z = s] \leq \min_{\{s'|s \leq s' \leq t\}} \left\{ E[y|\, z = s'] + \min_{\{u|u<s'\}} \frac{E[y|\, z = s'] - E[y|\, z = u]}{s' - u}(t - s') \right\}, \tag{5}$$

and for $t \leq s$,

$$E[y(t)|\, z = s] \geq \max_{\{s'|s \geq s' \geq t\}} \left(E[y|\, z = s'] + \max_{\{u|u<t\}} \left\{ \frac{E[y|\, z = s'] - E[y|\, z = u]}{s' - u}(t - s') \right\} \right). \tag{6}$$

Manski and Pepper (2000) showed that the MTR-MTS assumption implies that for $t > s$,

$$E\left[y(t)|\, z = s\right] \leq E\left[y|\, z = t\right],$$

and for $t \leq s$,

$$E\left[y(t)|\, z = s\right] \geq E\left[y|\, z = t\right].$$

These bounds are included in the case of $s' = t$ in Equations (5) and (6).
Applying Equations (5) and (6) to the Law of Iterated Expectations yields the second terms of the upper and lower bounds on $E\left[y(t)\right]$ in the Proposition, respectively.
Manski (1997) and Manski and Pepper (2000) showed that under either concave-MTR or MTS-MTR assumptions, for $s \geq t$,

$$E\left[y(t)|\, z = s\right] \leq E\left[y|\, z = s\right], \tag{7}$$

and for $s < t$,

$$E\left[y(t)|\, z = s\right] \geq E\left[y|\, z = s\right]. \tag{8}$$

This implies the first terms of the upper and lower bounds on $E\left[y(t)\right]$ in the Proposition. Thus, these results yield the bounds on $E\left[y(t)\right]$ in the Proposition.

The proof of the sharpness of the bounds is provided in the Appendix.

Manski's (1997) and Manski and Pepper's (2000) bounds are included in the brackets of Equations (5) and (6), and Equations (7) and (8). Equations (5) and (6) take minimum and maximum of the objects within these brackets, respectively. Therefore, the bounds in the Proposition are narrower than or equal to those of Manski (1997) and Manski and Pepper (2000). ■

The introduction of the assumption of concavity into the MTR and MTS assumptions narrows the width of the bounds on $E\left[y(t)\right]$ by,

$$\sum_{s \geq t} \left\{ \max_{\{s'|s \geq s' \geq t\}} \left(E\left[y|\, z = s'\right] + \max_{\{u|u<t\}} \left\{ \frac{E\left[y|\, z = s'\right] - E\left[y|\, z = u\right]}{s' - u} \left(t - s'\right) \right\} \right) - E\left[y|\, z = t\right] \right\}$$
$$\times P\left(z = s\right)$$

$$+ \sum_{s \leq t} \left(E\left[y|\, z = t\right] - \min_{\{s'|s \leq s' \leq t\}} \left\{ E\left[y|\, z = s'\right] + \min_{\{u|u<s'\}} \frac{E\left[y|\, z = s'\right] - E\left[y|\, z = u\right]}{s' - u} \left(t - s'\right) \right\} \right)$$
$$\times P\left(z = s\right). \tag{9}$$

The first term shows the increase in the lower bound and the second term shows the decrease in the upper bound.

This proposition implies the sharp upper bound on the average treatment effect, $E[y(t_2)] - E[y(t_1)]$ for $t_1 < t_2$, which is denoted by $\triangle(t_1, t_2)$, namely,

$$\begin{aligned}
&\triangle(t_1, t_2) \\
\leq\ & \sum_{s>t_2} E[y|\, z = s]\, P(z = s) \\
&+ \sum_{s\leq t_2} \min_{\{s'|s\leq s'\leq t_2\}} \left\{ E[y|\, z = s'] + \min_{\{u|u<s'\}} \frac{E[y|\, z = s'] - E[y|\, z = u]}{s' - u} (t_2 - s') \right\} P(z = s) \\
&- \sum_{s<t_1} E[y|\, z = s]\, P(z = s) \\
&- \sum_{s\geq t_1} \max_{\{s'|s\geq s'\geq t_1\}} \left(E[y|\, z = s'] + \max_{\{u|u<t_1\}} \left\{ \frac{E[y|\, z = s'] - E[y|\, z = u]}{s' - u} (t_1 - s') \right\} \right) P(z = s).
\end{aligned}$$

The upper bound on $\triangle(t_1, t_2)$ is nonnegative and sharp since the bounds in the Proposition are sharp; thus, $E[y(t_2)]$ and $E[y(t_1)]$ can take their upper and lower bounds, respectively.

3 Application to the Returns to Schooling

We use the 2000 wave of the National Longitudinal Survey of Youth (NLSY), which is representative of the U.S. non-institutionalized civilian population between the ages of 14 and 22 in 1979. As Manski and Pepper (2000), a random sample of white men is utilized; who reported that they were full-time, year-round workers and not self-employed. The sample size is 1240. Their hourly rate of pay and realized years of schooling were observed. In our application to the returns to schooling, z represents the realized years of schooling, the response variable $y_j(t)$ is the logarithm of hourly rate of pay a person j would obtain if he were to have t years of schooling, and $y_j = y_j(z)$ is the logarithm of the observed hourly wage.[1]

[1] We exclude three individuals whose wages are less than one dollar. Thus, the support of Y is $[0, \infty]$. Due to the small sample size, we exclude seven individuals who have seven years of schooling following Manski and Pepper (2000). By including these seven observations, similar results are obtained.

Table 1 shows the estimates of $E(y|z)$ and $P(z)$ that are used to estimate the bounds. Forty-one percent of the NLSY respondents have 12 years of schooling and 18 percent have 16 years of schooling. For the most part, the estimates of $E(y|z)$ in Table 1 increase with z. There are, however, three dips, which do not work with the monotone treatment selection assumption. As in Manski and Pepper (2000), when we compute the uniform 95 percent confidence intervals for the estimates of $E(y|z)$, the intervals contain everywhere monotone functions. When the slope is estimated, $\{E[y|\, z = s'] - E[y|\, z = u]\} \diagup (s' - u)$, negative slopes caused by dips are ignored.[2]

Table 2 reports the estimates of the bounds on $E[y(t)]$ in the Proposition. The 5 percent and 95 percent bootstrap quantiles are also shown. For comparison, Table 2 also reports the estimates of the bounds using only the MTR and MTS (Manski and Pepper's (2000) bounds), and the estimates of the bounds using only the concave-MTR (Manski's (1997) bounds). The estimates of our bounds are much narrower than the estimates of Manski and Pepper (2000) and Manski (1997). Specifically, our lower bound estimates increase more for lower years of schooling, whereas our upper bound estimates decrease more for higher years of schooling, compared to the estimates of Manski and Pepper (2000). Equation (9) shows that when t is small, the first term (the increase in the lower bound) dominates the reduction in the width of the bounds, whereas when t is large, the second term (the decrease in the upper bound) dominates it.

Table 3 reports the estimates of the bounds on the average treatment effect, $\Delta(t-1, t)$ for $t = 9, ..., 20$. Our bounds are listed in Columns 1 and 2, the bounds using only MTR and MTS of Manski and Pepper (2000) in Columns 3 and 4, and the bounds using only the concave-MTR of Manski (1997) in Columns 5 and 6. Our upper bound estimates on the average treatment effect are in the range of $0.150 - 0.239$ for more than 11 years of schooling. The smallest of our upper bound estimates is 0.150 for $\Delta(13, 14)$ and 0.152 for $\Delta(14, 15)$ and $\Delta(15, 16)$. Our upper bound estimates are reduced by $31 - 61$ percent from those using only MTR and MTS from Manski and Pepper (2000), where the largest reduction is in $\Delta(15, 16)$. Our upper bound estimates are also reduced by $84 - 94$ percent from those

[2] Additionally, we take the following three approaches. (i) We exclude the dips of the estimates of $E(y|z)$. (ii) When $\{E[y|\, z = s'] - E[y|\, z = u]\} \diagup (s' - u)$ is negative, we replace $E(y|z = u)$ with the lower 95 percent confidence interval of the estimate, and (iii) we replace $E(y|z = s')$ with the upper 95 percent confidence interval of the estimate. The conclusion is not changed, even though the estimates are shown to be slightly wider than in Table 2.

using only concave-MTR from Manski (1997).

Card (1999) surveys the previous studies that use the OLS and IV to estimate the causal effect of education on earnings. The point-estimates on the returns to schooling for the US data are in the range of $0.052 - 0.132$.[3] Our estimates of the upper bound on $\Delta(t-1,t)$ for $t = 13, 14, 15, 16$ are close to the upper range of the point-estimates on the returns to schooling in previous studies. The upper bound estimates on $\Delta(12,16)$ in Table 3 implies that the completion of four-years of college yields, at most, an increase of 0.257 in mean log wage relative to the completion of high school. The average value of the year-by-year college education treatment effect is at most 0.064. This estimate falls in the lower range of the point estimates of previous research.

4 Conclusion

The sharp bounds on the mean treatment response is obtained when the assumption of a concave response function is introduced to Manski and Pepper's (2000) assumption of monotone treatment response and monotone treatment selection. The empirical application to the returns to schooling shows that the estimates of our bounds become substantially smaller than those of Manski and Pepper (2000), and closer to the point estimates of previous empirical studies.

[3] The OLS estimates tend to be lower than the IV estimates.

5 Appendix: Proof of the Sharp Bounds in the Proposition

In order to show that the bounds are sharp, it suffices to show that the concave-MTR and MTS functions of $y_j(\tau)$ for $\tau \in T$ attain lower and upper bounds.

1. It is possible to take the concave-MTR and MTS functions of $y_j(\tau)$ for $\tau \in T$ that attain the *upper* bound.

Denote,

$$UB(s,t) = \min_{\{s'|s \le s' \le t\}} \left\{ E[y|\, z = s'] + \min_{\{u|u<s'\}} \frac{E[y|\, z = s'] - E[y|\, z = u]}{s' - u}(t - s') \right\}.$$

For $s < t$, take,

$$E[y(\tau)|\, z = s] = \min\left(\min_{\{\widetilde{s}|s \le \widetilde{s} < t\}} \left\{ E[y|\, z = \widetilde{s}] + \frac{UB(\widetilde{s},t) - E[y|\, z = \widetilde{s}]}{t - \widetilde{s}}(\tau - \widetilde{s}) \right\},\; E[y|\, z = t] \right),$$

for $\tau \in T$.

For $s \ge t$, take,

$$E[y(\tau)|\, z = s] = E[y|\, z = s],$$

for $\tau \in T$.

These functions attain upper bounds in the Proposition since $UB(s,t)$ and $E[y|\, z = s]$ are weakly increasing in s, and $E[y|\, z = s] \le UB(s,t) \le E[y|\, z = t]$ for $s \le t$.

$E[y(\tau)|\, z = s]$ satisfies the concave-MTR, since by definition its graph is the boundary of the convex hull, that is, the intersection of the subgraphs of the weakly increasing linear functions in τ: $E[y|\, z = \widetilde{s}] + \frac{UB(\widetilde{s},t) - E[y|z=\widetilde{s}]}{t - \widetilde{s}}(\tau - \widetilde{s})$ for $s \le \widetilde{s} < t$ and $E[y|\, z = t]$.[4] $E[y(\tau)|\, z = s]$ also satisfies the MTS, since by definition one takes minimum over the set of $\{\widetilde{s}\,|s \le \widetilde{s} < t\}$ such that $\{\widetilde{s}\,|s_1 \le \widetilde{s} < t\} \supseteq \{\widetilde{s}\,|s_2 \le \widetilde{s} < t\}$ for $s_1 \le s_2$.

We show $E[y(s)|\, z = s] = E[y|\, z = s]$.

First, for $\zeta \le \widehat{s} < t$,

$$E[y|\, z = \zeta] \le E[y|\, z = \widehat{s}] + \min_{\{u|u<\widehat{s}<t\}} \left\{ \frac{E[y|\, z = \widehat{s}] - E[y|\, z = u]}{\widehat{s} - u} \right\}(\zeta - \widehat{s}),$$

[4] The subgraph of $f(\tau)$ is defined as $\{(\tau, y)|\, y \le f(\tau)\}$.

since

$$E[y|z=\zeta] = E[y|z=\widehat{s}] + \frac{E[y|z=\widehat{s}] - E[y|z=\zeta]}{\widehat{s}-\zeta}(\zeta - \widehat{s})$$
$$\leq E[y|z=\widehat{s}] + \min_{\{u|u<\widehat{s}<t\}}\left\{\frac{E[y|z=\widehat{s}] - E[y|z=u]}{\widehat{s}-u}\right\}(\zeta - \widehat{s}).$$

Second,

$$\min_{\{u|u<s<t\}} \frac{E[y|z=s] - E[y|z=u]}{s-u} \geq \frac{UB(s,t) - E[y|z=s]}{t-s},$$

since

$$E[y|z=s] + \min_{\{u|u<s<t\}} \frac{E[y|z=s] - E[y|z=u]}{s-u}(t-s) \geq UB(s,t).$$

Therefore, for $s \leq \widetilde{s} < t$,

$$E[y|z=s] \leq E[y|z=\widetilde{s}] + \frac{UB(\widetilde{s},t) - E[y|z=\widetilde{s}]}{t-\widetilde{s}}(s-\widetilde{s}).$$

Thus,

$$E[y(s)|z=s] = E[y|z=s].$$

2. It is possible to take the concave-MTR and MTS functions of $y_j(\tau)$ for $\tau \in T$ that attain the *lower* bound.

Denote $s'^*(s,t)$ and $u^*(s,t)$ as the solutions of,

$$\max_{\{s'|s\geq s'\geq t\}}\left(E[y|z=s'] + \max_{\{u|u<t\}}\left\{\frac{E[y|z=s'] - E[y|z=u]}{s'-u}(t-s')\right\}\right).$$

Denote,

$$LB(\tau,s,t) \equiv \min_{\{\widetilde{s}|\widetilde{s}\geq s\}} \min\left\{\begin{array}{c} E[y|z=s'^*(\widetilde{s},t)] + \frac{E[y|z=s'^*(\widetilde{s},t)]-E[y|z=u^*(\widetilde{s},t)]}{s'^*(\widetilde{s},t)-u^*(\widetilde{s},t)}[\tau - s'^*(\widetilde{s},t)], \\ E[y|z=\widetilde{s}] \end{array}\right\}.$$

For $s \geq t$, take $E[y(\tau)|z=s] = LB(\tau,s,t)$.

For $s < t$, take,

$$E[y(\tau)|z=s] = \min\{E[y|z=s], LB(\tau,t,t)\}.$$

$E[y(\tau)|z=s]$ satisfies the concave-MTR, since its graph is the boundary of the convex hull, and weakly increasing in τ. $E[y(\tau)|z=s]$ also satisfies MTS, since one takes minimum over the set of $\{\widetilde{s}|\widetilde{s}\geq s\}$ such that $\{\widetilde{s}|\widetilde{s}\geq s_1\} \supseteq \{\widetilde{s}|\widetilde{s}\geq s_2\}$ for $s_1 \leq s_2$, and $E[y|z=s]$ is weakly increasing in s.

These functions attain the lower bounds in the Proposition for the following reason. First, for $\widetilde{s} \geq s$,

$$
\begin{aligned}
& E\left[y \mid z=s'^{*}(\widetilde{s}, t)\right]+\frac{E\left[y \mid z=s'^{*}(\widetilde{s}, t)\right]-E\left[y \mid z=u^{*}(\widetilde{s}, t)\right]}{s'^{*}(\widetilde{s}, t)-u^{*}(\widetilde{s}, t)}\left[t-s'^{*}(\widetilde{s}, t)\right] \\
\geq \; & E\left[y \mid z=s'^{*}(s, t)\right]+\frac{E\left[y \mid z=s'^{*}(s, t)\right]-E\left[y \mid z=u^{*}(s, t)\right]}{s'^{*}(s, t)-u^{*}(s, t)}\left[t-s'^{*}(s, t)\right],
\end{aligned}
$$

because of the definitions of $s'^{*}(\widetilde{s}, t)$ and $u^{*}(\widetilde{s}, t)$. Second,

$$
E[y \mid z=\widetilde{s}] \geq E\left[y \mid z=s'^{*}(\widetilde{s}, t)\right]+\frac{E\left[y \mid z=s'^{*}(\widetilde{s}, t)\right]-E\left[y \mid z=u^{*}(\widetilde{s}, t)\right]}{s'^{*}(\widetilde{s}, t)-u^{*}(\widetilde{s}, t)}\left[t-s'^{*}(\widetilde{s}, t)\right],
$$

since $E[y \mid z=\widetilde{s}]$ is weakly increasing in $\widetilde{s}$, $\widetilde{s} \geq s'^{*}(\widetilde{s}, t)>u^{*}(\widetilde{s}, t)$ and $t \leq s'^{*}(\widetilde{s}, t)$.

Therefore, for $s \geq t$,

$$
\begin{aligned}
E[y(t) \mid z=s] &= LB(t, s, t) \\
&= E\left[y \mid z=s'^{*}(s, t)\right]+\frac{E\left[y \mid z=s'^{*}(s, t)\right]-E\left[y \mid z=u^{*}(s, t)\right]}{s'^{*}(s, t)-u^{*}(s, t)}\left[t-s'^{*}(s, t)\right],
\end{aligned}
$$

and for $s<t$,

$$
E[y(t) \mid z=s]=E[y \mid z=s]
$$

by the definitions of $s'^{*}(t, t)$ and $u^{*}(t, t)$.

It suffices to show that $E[y(s) \mid z=s]=E[y \mid z=s]$.

The definitions of $s'^{*}(\widetilde{s}, t)$ and $u^{*}(\widetilde{s}, t)$ and the fact that for $\widetilde{s} \geq s \geq t$

$$
\begin{aligned}
& E\left[y \mid z=s'^{*}(\widetilde{s}, t)\right]+\frac{E\left[y \mid z=s'^{*}(\widetilde{s}, t)\right]-E\left[y \mid z=u^{*}(\widetilde{s}, t)\right]}{s'^{*}(\widetilde{s}, t)-u^{*}(\widetilde{s}, t)}\left[t-s'^{*}(\widetilde{s}, t)\right] \\
= \; & E\left[y \mid z=u^{*}(\widetilde{s}, t)\right]+\frac{E\left[y \mid z=s'^{*}(\widetilde{s}, t)\right]-E\left[y \mid z=u^{*}(\widetilde{s}, t)\right]}{s'^{*}(\widetilde{s}, t)-u^{*}(\widetilde{s}, t)}\left[t-u^{*}(\widetilde{s}, t)\right] \\
\geq \; & E\left[y \mid z=u^{*}(\widetilde{s}, t)\right]+\frac{E[y \mid z=s]-E\left[y \mid z=u^{*}(\widetilde{s}, t)\right]}{s-u^{*}(\widetilde{s}, t)}\left[t-u^{*}(\widetilde{s}, t)\right]
\end{aligned}
$$

imply that for $\widetilde{s} \geq s \geq t$

$$
\frac{E\left[y \mid z=s'^{*}(\widetilde{s}, t)\right]-E\left[y \mid z=u^{*}(\widetilde{s}, t)\right]}{s'^{*}(\widetilde{s}, t)-u^{*}(\widetilde{s}, t)} \geq \frac{E[y \mid z=s]-E\left[y \mid z=u^{*}(\widetilde{s}, t)\right]}{s-u^{*}(\widetilde{s}, t)} \geq 0.
$$

Thus,

$$
\begin{aligned}
& E\left[y\right| z=s'^{*}(\widetilde{s},t)]+\frac{E\left[y\right| z=s'^{*}(\widetilde{s},t)]-E\left[y\right| z=u^{*}(\widetilde{s},t)]}{s'^{*}(\widetilde{s},t)-u^{*}(\widetilde{s},t)}\left[s-s'^{*}(\widetilde{s},t)\right] \\
= \;& E\left[y\right| z=u^{*}(\widetilde{s},t)]+\frac{E\left[y\right| z=s'^{*}(\widetilde{s},t)]-E\left[y\right| z=u^{*}(\widetilde{s},t)]}{s'^{*}(\widetilde{s},t)-u^{*}(\widetilde{s},t)}\left[s-u^{*}(\widetilde{s},t)\right] \\
\geq \;& E\left[y\right| z=u^{*}(\widetilde{s},t)]+\frac{E\left[y\right| z=s]-E\left[y\right| z=u^{*}(\widetilde{s},t)]}{s-u^{*}(\widetilde{s},t)}\left[s-u^{*}(\widetilde{s},t)\right] \\
= \;& E\left[y\right| z=s] \quad \text{for } \widetilde{s}\geq s\geq t.
\end{aligned}
$$

Hence, $E\left[y(s)\right| z=s] = LB(s,s,t) = E\left[y\right| z=s]$ for $s \geq t$. Similarly, $LB(s,t,t) = E\left[y\right| z=t]$. Therefore, $E\left[y(s)\right| z=s] = E\left[y\right| z=s]$ for $s < t$.

References

[1] Card, D, 1999, Casual Effect of Education on Earnings, In *Handbook of Labor Economics, Vol. 3A.* Ashenfelter, O.C. and D. Card (eds.). North-Holland, Amsterdam.

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Table 1: Mean Log(Wages) and Distribution of Schooling: Wave 2000, NLSY

z	$E(y \mid z)$	$P(z)$	Sample Size
8	2.541	0.016	20
9	2.449	0.019	23
10	2.515	0.017	21
11	2.637	0.019	24
12	2.715	0.410	508
13	2.985	0.071	88
14	2.979	0.088	109
15	3.062	0.037	46
16	3.244	0.182	225
17	3.266	0.037	46
18	3.381	0.051	63
19	3.358	0.025	31
20	3.368	0.029	36
Total		1	1240

Table 2: Upper and Lower Bounds on E[*y* (*t*)]

t	Our Lower Bound on E[*y* (*t*)]			Our Upper Bound on E[*y* (*t*)]		
	Estimate	0.05 Bootstrap Quantile	0.95 Bootstrap Quantile	Estimate	0.05 Bootstrap Quantile	0.95 Bootstrap Quantile
8	2.541	2.386	2.696	2.943	2.917	2.968
9	2.602	2.509	2.707	2.941	2.915	2.967
10	2.668	2.608	2.757	2.944	2.916	2.969
11	2.732	2.695	2.803	2.947	2.921	2.973
12	2.796	2.768	2.843	2.951	2.923	2.976
13	2.842	2.819	2.888	2.972	2.937	2.998
14	2.871	2.852	2.907	2.992	2.944	3.025
15	2.901	2.880	2.933	3.024	2.956	3.060
16	2.929	2.905	2.957	3.053	2.969	3.097
17	2.937	2.912	2.965	3.085	2.978	3.136
18	2.944	2.917	2.971	3.117	2.994	3.178
19	2.943	2.917	2.969	3.148	2.997	3.209
20	2.943	2.917	2.968	3.182	3.004	3.233

t	Manski and Pepper's Lower Bound on E[*y* (*t*)]			Manski and Pepper's Upper Bound on E[*y* (*t*)]		
	Estimate	0.05 Bootstrap Quantile	0.95 Bootstrap Quantile	Estimate	0.05 Bootstrap Quantile	0.95 Bootstrap Quantile
8	2.541	2.386	2.696	2.943	2.917	2.968
9	2.451	2.309	2.603	2.941	2.915	2.967
10	2.514	2.302	2.686	2.944	2.916	2.969
11	2.630	2.514	2.746	2.950	2.923	2.976
12	2.702	2.669	2.734	2.955	2.929	2.981
13	2.842	2.799	2.887	3.085	3.038	3.132
14	2.840	2.801	2.879	3.082	3.027	3.132
15	2.870	2.827	2.913	3.135	3.062	3.211
16	2.929	2.900	2.955	3.258	3.212	3.304
17	2.932	2.902	2.961	3.277	3.161	3.393
18	2.944	2.916	2.970	3.380	3.274	3.478
19	2.943	2.916	2.967	3.358	3.202	3.511
20	2.943	2.917	2.968	3.368	3.226	3.514

t	Manski's Lower Bound on E[*y* (*t*)]			Manski's Upper Bound on E[*y* (*t*)]		
	Estimate	0.05 Bootstrap Quantile	0.95 Bootstrap Quantile	Estimate	0.05 Bootstrap Quantile	0.95 Bootstrap Quantile
8	0.508	0.493	0.523	2.943	2.917	2.968
9	0.974	0.956	0.993	2.984	2.955	3.012
10	1.418	1.397	1.440	3.048	3.010	3.086
11	1.848	1.823	1.872	3.125	3.075	3.178
12	2.265	2.237	2.293	3.216	3.151	3.286
13	2.460	2.434	2.484	3.529	3.450	3.613
14	2.619	2.596	2.641	3.878	3.784	3.977
15	2.741	2.718	2.762	4.263	4.153	4.379
16	2.848	2.824	2.871	4.663	4.535	4.796
17	2.891	2.866	2.914	5.129	4.985	5.277
18	2.921	2.895	2.946	5.606	5.447	5.771
19	2.935	2.910	2.960	6.099	5.925	6.280
20	2.943	2.917	2.968	6.599	6.412	6.796

Table 3: Upper Bounds on Returns to Schooling

		Upper Bounds on $\Delta(s, t)$					
		Our Estimate		Manski and Pepper's Estimate		Manski's Estimate	
		Estimate	0.95 Bootstrap Quantile	Estimate	0.95 Bootstrap Quantile	Estimate	0.95 Bootstrap Quantile
s	***t***	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
8	9	0.400	0.557	0.400	0.557	2.476	2.503
9	10	0.341	0.437	0.493	0.638	2.073	2.107
10	11	0.279	0.340	0.436	0.644	1.707	1.755
11	12	0.219	0.254	0.326	0.443	1.368	1.435
12	13	0.176	0.204	0.383	0.440	1.264	1.347
13	14	0.150	0.182	0.240	0.307	1.418	1.515
14	15	0.152	0.183	0.296	0.383	1.644	1.756
15	16	0.152	0.189	0.389	0.451	1.923	2.052
16	17	0.156	0.202	0.349	0.466	2.280	2.426
17	18	0.181	0.238	0.449	0.549	2.716	2.879
18	19	0.205	0.262	0.414	0.570	3.179	3.359
19	20	0.239	0.290	0.426	0.576	3.664	3.861
12	16	0.257	0.308	0.556	0.612	2.398	2.522
averege effect		0.064	0.077	0.139	0.153	0.600	0.630