

Economics 7800  
December 21, 1998  
Final Exam

Please answer each question in as much detail as necessary to provide a complete answer, but without gratuitous padding. You will have no more than 2 hours minutes to finish the exam. Good luck.

1. In class, we derived relationships among housing production, land rents and housing prices such that:

$$\frac{\eta \ln(K(u)/L(u))}{\eta} = \mathbf{S} \frac{\eta \ln R(u)}{\eta} = \frac{\mathbf{S}}{\mathbf{r}_L} \frac{\eta \ln p(u)}{\eta}$$

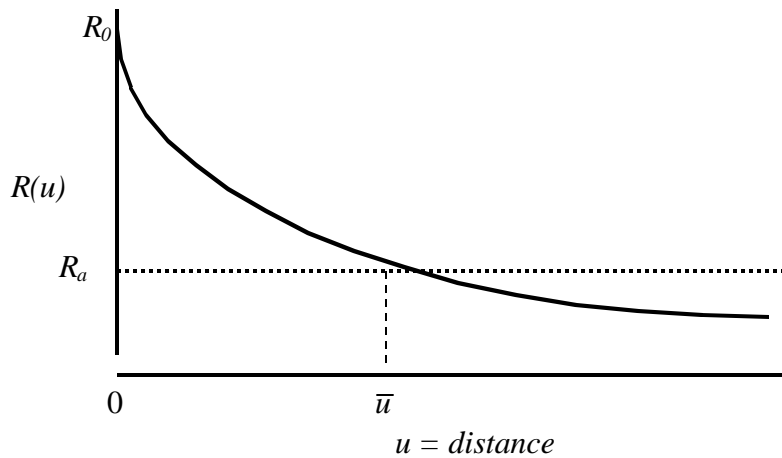
where  $\mathbf{S}$  is the elasticity of substitution and  $\mathbf{r}_L$  is the land share of the housing price.  $u$  is distance from the central place,  $K$  is capital, and  $L$  is land.

Suppose that housing is constructed with the production function:

$$Q = K^{0.9}L^{0.1}$$

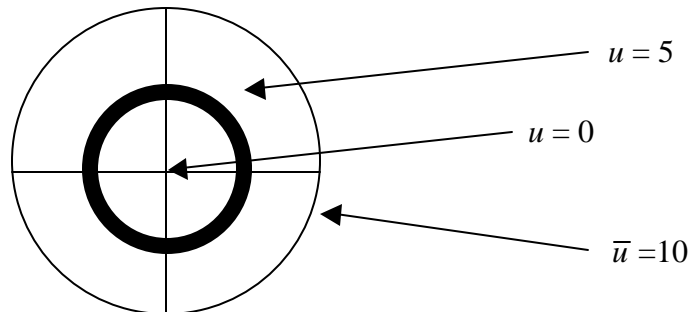
- Calculate the elasticity of substitution of capital for land.
- What is the relationship between land rent and the capital-labor ratio?
- What is the relationship between housing price and land rent? Why?

2. Consider land rents in an urban area such that:



Use the above diagram to aid your analyses. Let  $R_a$  equal the agricultural rent, such the  $\bar{u}$  is the edge of the city.  $R_0$  is the rent in the Central Business District.

a. Assume an open city, and let  $\bar{u}$  equal 10. Suppose that there is an amenity that occurs at various levels throughout the city. Let the amenity increase only at  $u = 5$ , as noted in the dia-gram below. Discuss what will happen to levels of utility and to land rents throughout the city.



b. Assume, now, a closed city and let  $\bar{u}$  equal 10. Suppose again that there is an amenity that occurs at various levels throughout the city. Let the amenity increase at all distances greater than  $u = 5$ . What will happen to the utility levels of the population? What will happen to land rents at various locations, and to aggregate (the sum of all of the) land rents in the city?

c. Consider an open city, and assume that the consumer's utility function is:

$$V = \text{goods}^a \text{land}^b \text{amentities}^d$$

where  $a + b = 1$ . The income constraint is:

$$y = \text{goods} + (\text{price of land} * \text{land}) + \text{commuting costs}.$$

Derive briefly, and explain fully, the equilibrium rent schedule for the price of land.

3. Consider the following demand for land function:

$$Land = 110 - .1 * Land\ rent,$$

The market price of land is 100. However, land use pollutes the stream, leading to a social cost function of:

$$Social\ Cost = 100 + .01 * Land.$$

- a. Calculate the market equilibrium usage of land, ignoring externalities. Calculate the optimal use.
- b. Calculate the social loss occurring due to the externality.
- c. Calculate the land tax that would be necessary to promote the optimal use of land. What would be done with this tax once collected.

4. Goodman and Kawai (1982) estimated the following housing demand regression.

$$Q = 3.057 + 0.213 * \text{Permanent Income} + 0.082 * \text{Transitory Income} - 0.029 * \text{Housing Price} + \text{Assorted Other Variables.}$$

- a. Why do housing economists feel that it is important to use permanent rather than observed income, in order to measure housing demand?
- b. Assuming an income of 12 (thousand), a price of 100. and a predicted quantity of 2.72 housing units, calculate the permanent income elasticity of housing demand, and the price elasticity of housing demand.
- c. Derive the impact of an increase in income on the price elasticity of demand. Explain your results.

5. Housing market discrimination has had an important role in urban analysis.
  - a. Describe briefly three models of housing discrimination with respect to race.
  - b. How do racial segregation and discrimination relate to each other? How do they differ?
  - c. If you conduct a very detailed hedonic price study to test for discrimination, how would you do it. Be very specific as to which variables you would include, and how you would interpret them.