Vaccination Externalities

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Abstract

Vaccination provides indirect benefits to the unvaccinated. Despite its important policy implications, there is little analytical or empirical work to quantify this externality, nor is it incorporated in a number of cost-benefit studies of vaccine programs. We use a standard epidemiological model to analyze how the magnitude of this externality varies with the number of vaccinations, vaccine efficacy, and disease infectiousness. We also provide empirical estimates using parameters for influenza and mumps epidemics. The pattern of the externality is complex and striking, unlike that suggested in standard treatments. The size of the externality is not necessarily monotonic in the number vaccinated, vaccine efficacy, nor disease infectiousness. Moreover, its magnitude can be remarkably large. In particular, the marginal externality of a vaccination can be greater than one case of illness prevented among the nonvaccinated, so its omission from policy analyses implies serious biases.

KEYWORDS: externality, vaccination, public health, epidemic, influenza, mumps, epidemiological models

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1. Introduction

The optimal vaccination level is where the marginal social benefit of the last vaccination equals its marginal cost. The marginal social benefit of a vaccination includes the direct health benefits to the person inoculated, plus the indirect benefits to those not inoculated, since vaccinated individuals are less likely to infect others. Economists have paid great attention to the role of these externalities in discussing vaccination policies (Folland et al., 1993, pp. 436-438; Hemenway, 1994; Ekelund and Ault, 1995, pp. 587-588; Phelps, 1997, pp. 502-512; Browning and Zupan, 1999, p. 542; Kremer, 2000; Office of Health Economics, 2002, p. 30; Santerre and Neun, 2004, p. 233; and Henderson, 2005, p. 75).

Actual policies pay much less attention to externalities. Consider, for example, the most recent influenza vaccination recommendations of the Advisory Committee on Immunization Practices (N. Smith et al., 2006, p. 2). Recommendations of whom to vaccinate focus on persons at high risk for influenza-related complications or death (e.g., children aged 6 to 59 months, adults and children with chronic pulmonary disorders, and adults aged 50 and over). The rationale for these recommendations does not take into account some research that indicates a preferred strategy might be to target sub-populations, such as school children, that are most likely to generate negative externalities by spreading the disease (Ackerman et al., 1984; pp. 144-146; Halloran, 2001; and Reichert et al. 2001). Moreover, there is no discussion of the optimal fraction of the population to be vaccinated. Similarly, the Centers for Disease Control (CDC) reports on the economic impacts of pandemic influenza (Meltzer et al., 1999a and 1999b) ignore externalities in calculating benefits and costs of different levels of vaccination against pandemic flu. If externalities are important, then their estimates of the return to vaccination are understated.

This lack of attention to vaccine externalities in policy discussions is echoed in benefit-cost and cost-effectiveness studies, which often exclude externality effects. Where adjustments for externalities are made, investigators commonly assume they are a constant multiple of benefits for vaccinees. Statistical estimates of vaccination effectiveness in reducing the incidence of disease have also ignored these indirect effects of vaccination.

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1The ACIP is a committee of experts appointed by the Secretary of Health and Human Services to advise the Secretary and the CDC on vaccination policy.

2Examples from the literature on influenza include Schoenbaum (1987); Perez-Tirse and Gross (1992); Campbell and Rumley (1997); Bridges et al., (2000), and Nichol et al. (1995). Perez-Tirse and Gross (1992, p. 203) and Bridges et al. (2000, p. 1662) note the existence of externalities but do not quantify them.

3For example, the World Bank’s evaluation of an immunization program for children under age one in low-income countries multiplied benefits accruing to vaccinated infants by 1.2 to account for externalities among unvaccinated older children (Cowley et al. 1993). The program included immunization for diphtheria, pertussis, tetanus, measles, oral polio, and tuberculosis in all low-income countries plus hepatitis B and yellow fever in selected countries.

4Examples from the influenza literature include Campbell and Rumley (1997), Bridges et al. (2000), and Nichol et al. (1995).
If externalities are small, there are no consequences to ignoring them. If they are large, however, they may be worth trying to estimate and to include in policy analysis. Indeed, if externalities are sizable, then ignoring them may seriously bias estimates of the benefits and costs of vaccination programs, cost-effectiveness comparisons of alternative programs, and measures of the effectiveness of vaccination in preventing disease.

So, are externalities large or small, and what do they depend on? Surprisingly, there is little analytical or empirical work to quantify the magnitude of vaccination externalities. This paper helps to fill this gap. We use a standard epidemiological model (the Susceptible-Infectious-Removed or SIR model) to provide what we believe is the first analytical treatment showing how the magnitude of marginal social benefits and externalities from vaccination vary with the number of vaccinations, vaccine efficacy, and infectiousness of the disease.

There are a variety of applications of the SIR model in the economics literature. Francis (1997, 2004) provides theoretical examinations of optimal taxes and subsidies in vaccination markets and extends the seminal paper by Brito, Sheshinski, and Intrilligator (1991) by replacing their *ad hoc* specification of the effects of vaccination on the probability of disease transmission with those generated from a SIR model. Unlike our paper, Francis does not investigate the magnitudes or determinants of the marginal social benefits and externalities associated with vaccination, and he assumes that vaccination is 100% effective in preventing disease. Geoffard and Philipson (1997) examine whether the private market has adequate incentive to eradicate a disease. Kremer and Snyder (2006) examine whether there are market biases that lead firms to invest resources in producing drugs for treatment of disease rather than vaccines for prevention. Boulier (2006) considers the effects of monopoly and monopoly regulation on the price of vaccine and the quantity of vaccines produced and consumed and compares these outcomes with the socially optimal level of provision.

Our analysis finds that the pattern of variation in magnitude of externalities is more complicated than prior discussions have suggested. In particular, the patterns of marginal social benefits and externalities differ markedly from textbook depictions. We provide simulations of vaccination externalities, primarily for influenza epidemics, but also for mumps, to illustrate the principal findings from our analytical work.

There are three reasons for the primary focus on influenza. First, influenza is an important disease. According to the Centers for Disease Control (2002), about 10%-20% of the U.S. population have the flu each year, and about 114,000 hospitalizations and 36,000 deaths (W. Thompson et al. 2003, p. 182) are attributed to influenza annually. Second, there is an effective vaccination for influenza. Third, influenza outbreaks are relatively well described by the epidemiological model we use. Transmission of the disease is by person-to-person contact, so that we can ignore the complications introduced by intermediate vectors such as mosquitos in the case of yellow fever. Another advantage of examining influenza is that epidemics are of short

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According to Palache (1997, p. 844), influenza vaccination is 70%-90% effective in preventing influenza among adults less than age 60 and 50%-80% effective among adults over age 60. Palache (p. 843) also states that U.S. Army studies find an effectiveness rate of 70-90% for armed services personnel.
duration, so that it is not unreasonable to assume (as we do) that population size is fixed, allowing us to ignore the dynamics of endemic diseases. Our comparative results for mumps can only be viewed as illustrative of a disease with higher infectivity, since births provide a fresh supply of susceptibles, so that the disease is endemic. A dynamic model incorporating population growth is required to fully capture the characteristics of this disease.

Section 2 introduces the Susceptible-Infectious-Removed (SIR) model of epidemics. Section 3 uses the SIR model to examine the determinants of the marginal social benefits of vaccination, measured as the number of illnesses prevented by an additional vaccination. It shows how these benefits of vaccination depend upon the extent of vaccination, vaccine efficacy, and the infectiousness of the disease. Section 4 examines the demand curve for vaccinations and the determinants of the marginal private benefits of vaccination. Section 5 presents an analysis of the marginal externalities of vaccination, which equal the difference between marginal social and private benefits. Section 6 presents conclusions, including the implications of our findings for cost-benefit and cost-effectiveness studies of vaccination, measurement of the effectiveness of vaccination, and vaccination policy.

There are several interesting results from the theoretical and empirical analyses. First, marginal social benefits of vaccination may either increase or decrease with the fraction of the population vaccinated. Second, vaccination externalities can be quite large. If the fraction of the population that is susceptible is sufficiently large and the disease is sufficiently infectious, the marginal externality of a vaccination can exceed one case of disease prevented among the unvaccinated. Third, the sizes of marginal externalities are not constant and they may rise and then fall with an increase in the number of vaccinations. We show how the exact pattern depends on underlying parameters. Fourth, we show that the marginal externality may either rise or fall with the efficacy of the vaccine and the infectiousness of the disease and show explicitly how the directions of change depend on the parameters of the model.

2. The Model

The standard deterministic model for non-recurring epidemics is the Susceptible-Infective-Removed (SIR) model developed by Kermack and McKendrick (1927). In this section, we introduce the basics of this theoretical model used in subsequent sections to derive the marginal social and private benefits of vaccination and the marginal externalities.

We have modified the Susceptible-Infective-Removed (SIR) model to include vaccination. The population is divided into four groups. Susceptibles (S) can catch the disease. Infectives (I) have the disease and can transmit it. The removed (R) are those who have recovered from the disease, are no longer infectious, and cannot be
reinfected. We assume that vaccinated individuals (V) are immunized prior to the outbreak of the disease and that m is the proportion of vaccinees who can neither catch nor transmit the disease. That is, m is a measure of the effectiveness of the vaccination. If m = 1, then vaccination is 100% effective. Individuals whose vaccinations are ineffective ((1-m)V) are included in the pool of susceptibles.

The total size of the population (N) is fixed and there are no births, deaths, or migration:

\[ N = S(t) + I(t) + R(t) + mV. \]  

(1)

Initial conditions are:

\[ S(0) = S_o = N - I_o - mV > 0, \quad I(0) = I_o > 0, \quad \text{and} \quad R(0) = 0. \]  

(2)

Note that an increase in the number or effectiveness of vaccinations reduces the size of the initially susceptible population \( S_o \).

The changes in the health of the population over time are given by a system of differential equations. The number of susceptibles declines because of new infections:

\[ \frac{dS(t)}{dt} = -\beta \frac{S(t)I(t)}{N}. \]  

(3)

Assuming the per period rate (\( \varphi \)) at which individuals contact others during an outbreak of disease is constant and that there is random contact among individuals in the population, then the number of new infections is given by \( \beta S(t)I(t)/N \), where the infection parameter \( \beta \) is the product of the contact rate and the probability of transmission of disease (\( \tau \)) given contact between an infective and a susceptible. Note that we have adopted the standard epidemiological modeling assumption that \( \varphi \) and \( \tau \) are exogenous. That is, individuals do not alter their probabilities of contact or preventive behaviors (e.g., condom use in the case of venereal disease) depending upon the prevalence of the disease. This assumption has been rightly criticized by Philipson (1995, pp. 218-219), who argues that individuals are likely to undertake preventive measures when the risk of becoming infected is high. However, his own applications of the SIR model (Philipson, 1996; Geoffard and Philipson, 1997) use the same simplifying assumption adopted here.

The number of infectives is augmented by the number of newly infected and decreases with the number who are removed (i.e., recover):

\[ \frac{dI}{dt} = \beta \frac{S(t)I(t)}{N} - \gamma I(t). \]  

(4)

where \( \gamma \) is the removal rate. This formulation assumes that the incubation period of the disease is negligible and that there is a constant hazard rate (\( \gamma \)) for exiting the infective class. \( 1/\gamma \) is the duration of the infective period.

Finally, changes in the size of the removed class are given by

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8 In other applications the recovered class could include those who die from the disease. For some diseases (e.g., syphilis but not influenza), reinfection is possible. Models with reinfection are called Susceptible-Infectious-Susceptible (or SIS) models.

9 Previous analyses by economists using the SIR model have assumed that vaccination is 100% effective.
The fraction “never-infected” among the unvaccinated must be less than $1/\sigma$, since $i(t)$ continues to rise and $s(t)$ continues to fall as long as $s(t) > 1/\sigma$.

If $\beta S_o / N < \gamma I_o$, then the number of new infectives at the time of the introduction of the disease is less than the number of removals. Therefore, the number of infectives declines from the very beginning and there is no epidemic. This condition can be reexpressed as

$$S_o < \frac{\gamma N}{\beta} = \frac{N}{\sigma}, \text{ or}$$

$$S_o < \frac{1}{\sigma},$$

where the parameter $\sigma = \beta/\gamma$ is called the “contact number” and $S_o (= S_o / N)$ is the proportion of the population that is initially susceptible. The contact number is the average number of susceptibles infected by an individual who is infectious when the population is wholly susceptible. $\sigma$ has been estimated to be about 1.4 for influenza (Hethcote, 2001, p. 617) and from 4 to 7 for mumps (Fine, 1993, Table 1, p. 268).

From (7b), there is a critical fraction of the susceptible population ($s_c = 1/\sigma$), below which an epidemic fails to take hold. However, if $s_o > 1/\sigma$, then the fraction of the population infective ($i(t) = I(t)/N$) initially increases and there is an epidemic. The proportion of the population that is infective continues to increase until the fraction susceptible falls to $s(t) = 1/\sigma$. At this point, the fraction infective declines monotonically, with $i(\infty) = 0$.

When the disease ceases spreading, a positive fraction of susceptibles, $s(\infty)$, remains uninfected. It can be shown that

$$s(\infty) = s_o e^{-\sigma(\infty)},$$

where $r(\infty)$ is the fraction “ever-infected”. (See Hethcote, 2001, p. 607.) $r(\infty)$ equals $s_o + i_o - s(\infty)$ and is the positive root of the transcendental equation:

$$s_o + i_o - r(\infty) = s_o e^{-\sigma(\infty)}.$$

both sides of this equation equal $s(\infty)$. See Hethcote (2001, p. 607).
3. Determinants of the Marginal Social Benefits of Vaccinations

The marginal social benefit of a vaccination is the social value of the number of illnesses prevented by an additional vaccination. If we assume that the population is homogeneous and risk neutral and that all individuals incur a constant cost $k$ if infected, then the marginal social benefit of a vaccination is just $k$ times the marginal effect of a vaccination, defined as the number of illnesses prevented by an additional vaccination.\(^{12}\) Here we examine the determinants of the marginal effect of a vaccination and show how it varies with the number of vaccinations (Section 3.1) and the infectiousness of the disease (Section 3.2).

3.1 The Marginal Effect of a Vaccination and the Number of Vaccinations

The marginal effect of a vaccination ($MEV$) in the SIR model is given by:

$$MEV = -\frac{\partial R(\infty)}{\partial V}. \quad (10)$$

Differentiating (9) with respect to $V$ yields:

$$MEV = -\frac{\partial R(\infty)}{\partial V} = \frac{m(1 - e^{-\sigma s(\infty)})}{1 - s_0 e^{-\sigma s(\infty)}}. \quad (11)$$

There are two things to note about the marginal effect of a vaccination. First, it has a positive sign.\(^{13}\) That is, vaccinations, by reducing the size of the initially susceptible population, decrease the number of persons who eventually become ill. Second, if the fraction of the initially susceptible population in the absence of vaccination is sufficiently large, the level of vaccination is sufficiently small, or the efficacy of the vaccine is sufficiently low that the fraction of the population that is susceptible exceeds the critical level for an epidemic ($s_0 > 1/\sigma$), then each additional vaccination prevents more than $m$ additional cases of infection, where $m$ is the measure of vaccine efficacy. If the fraction of the population that is susceptible just equals this critical number ($s_0 = 1/\sigma$), an additional vaccination prevents $m$ illnesses. If, instead, the number of effective vaccinations is sufficiently large to reduce this fraction below the critical level at which an epidemic sets in ($s_0 < 1/\sigma$), then each additional vaccination prevents fewer than $m$ additional cases of disease.

An implication of this result is that the marginal effect of a vaccination is not constant. For example, if the susceptible population is initially large enough to generate an epidemic ($s_0 > 1/\sigma$), the marginal effect of the first vaccination is greater than $m$. As the number of vaccinations increases, the marginal effect of a vaccination rises to a peak, declines to equal $m$ when vaccinations reduce the susceptible population to $s_0 = 1/\sigma$, and then continues to fall with additional vaccinations. This finding that the marginal social benefit of a vaccination can increase with the number

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\(^{12}\)If individuals are risk averse, then there are additional benefits of vaccination from risk reduction.

\(^{13}\)The numerator is positive since $s_0 > s_0 e^{m(\infty)} = s(\infty)$ which gives $1 > e^{\sigma s(\infty)}$, and the denominator is positive since $s_0 e^{m(\infty)} = s(\infty) < (1/\sigma)$.
of vaccinations is new. There is no previous analysis that indicates or depicts this result.

How large is the maximum size of the marginal effect of a vaccination as it varies with the number of vaccinations? Maximizing (11) with respect to \( V \) determines the fraction of the population vaccinated at which an increase in the number vaccinated has a maximum effect on the number of cases prevented. It can be shown that the marginal effect of a vaccination is greatest when

\[
\begin{align*}
\sigma_o^* &= \frac{1}{\sigma} + \frac{r(\infty) - i_o}{2}.
\end{align*}
\]

The second term on the right-hand-side of (12) equals one-half of the fraction ever-infected minus the fraction initially infected and is necessarily positive. Hence, the effectiveness of a marginal vaccination is greatest when the fraction susceptible is greater than the critical size necessary to cause an epidemic (\( s_o = 1/\sigma \)).

The maximum marginal effect of a vaccination is obtained as follows. First, using (9) to substitute \( (s_o^* + i_o - r(\infty))/s_o^* \) for \( e^{\sigma r(\infty)} \) in (11) and rearranging yields an alternative expression for (10):

\[
MEV^* = \frac{m}{s_o^*}(\frac{r(\infty) - i_o}{r(\infty) + 1/\sigma - i_o - s_o^*}).
\]

Substituting (12) into (13) yields:

\[
MEV^* = \frac{2m}{\sigma s_o^*}.
\]

Since \( s_o^* > 1/\sigma \), the denominator of (14) must be greater than 1, so that the largest value of the marginal effect of a vaccination must be less than 2m.

**Example.** Figure 1 displays estimates of the marginal effects of vaccination for influenza (\( \sigma = 1.4 \)) and mumps (\( \sigma = 5.5 \)) with different levels of vaccination effectiveness for a population size of \( N = 1000 \) and 1 initial infective.\(^4\) Consider the marginal effects for influenza first. If no one is vaccinated, 513 or 51.3% of the population would become infected. If vaccination is 100% effective (\( m = 1 \)), the marginal effect of the first vaccination on the number of cases prevented is 1.61. The marginal effect increases with subsequent vaccinations, peaking at 1.77 for 194 vaccinations. The marginal effect of a vaccination declines sharply after the peak, dropping below 1.0 for the 286\(^{th} \) vaccination, when 71.4% (= 100% * 1/\( \sigma \)) of the population is susceptible. Note that if 30% of the population had immunity to this strain of influenza, because they possessed antibodies from a previous infection, then the marginal effect of the first vaccination would only equal .62, which equals the marginal effect of the 301\(^{st} \) vaccination when all individuals are susceptible. In this case, the first vaccination would have the highest marginal effect.

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\(^4\)The contact number for mumps is estimated to range from 4 to 7 (Fine, 1993). We have taken the average of these values (\( \sigma = 5.5 \)) for this illustration.
Figure 1. Marginal Effect of Vaccination for Influenza ($\sigma = 1.4$) and Mumps ($\sigma = 5.5$) for Vaccines of Differing Efficacy, $N = 1000$ and $I_0 = 1$
When vaccination is 80% effective \((m = .8)\),\(^{15}\) the marginal effects of a vaccination are initially smaller than when vaccination is 100% effective. The peak marginal effect of a vaccination is 1.42 cases of illness prevented for the 242\(^{nd}\) vaccination. The marginal effect for the 80% effective vaccine exceeds that of the 100% effective vaccine for 267 or more vaccinations. The marginal effect is greater in this range with the less effective vaccine, since there are (ineffectively) vaccinated individuals who are still potential sources of infection to the unvaccinated, so that the risk of infection to the unvaccinated remains high.

For mumps, with \(\sigma = 5.5\) and \(m = 1\), the peak marginal effect is 1.67 for the 781\(^{st}\) vaccination. 81.7% of the population must be vaccinated before the marginal effect of a vaccination drops below 1. The marginal effect of vaccination when \(m = .8\) exceeds the marginal effect of a perfect vaccine for 816 or more vaccinations.

3.2 The Marginal Effect of a Vaccination and Infectiousness

The contact number \(\sigma = \beta/\gamma\) is a summary measure of the infectiousness of a disease. It reflects three channels by which a disease could be characterized as “more infectious”. \(\beta\) depends on the rate of contact among members of the population (\(\phi\)) and the transmissibility of disease given contact (\(\tau\)). For example, because children are gathered in school in winter, the infectiousness of mumps is much higher in the winter than summer months reflecting the higher contact rate (\(\phi\)) in winter.\(^{16}\) Influenza is more infectious than Hansen’s disease (leprosy) because it has a higher transmission rate \(\tau\); a single casual contact can spread influenza while leprosy requires intimate contact over a long period of time. Finally, a disease is more infectious if those who have the disease are infective for a longer period of time (\(1/\gamma\)). Mumps is more infectious than influenza not only because it has a higher \(\beta\) (.46 vs. .31), but also because the average duration of the infectious period for mumps exceeds that of influenza (12 days vs. 4.1 days).

The analysis of an increase in infectiousness on the magnitude of the marginal effect of a vaccination is straightforward. At any given proportion of the population vaccinated, the marginal effect of a vaccination is zero when the disease is non-infectious, rises to reach a peak (given by equation 14) as \(\sigma\) increases, and then falls with further increases in \(\sigma\).

**Example.** Suppose that 19.4% of the population is vaccinated, so that 80.6% is susceptible. If \(\sigma = 0\), then the marginal effect of a vaccination is zero. As \(\sigma\) rises, the marginal effect reaches 1.0 when \(\sigma = 1.24\) (= 1/.806) and reaches a peak of 1.77 when \(\sigma = 1.4\). (See Figure 1 for influenza.) Further increases in \(\sigma\) result in declines in the maximum marginal effect of vaccination. For example, if \(\sigma = 5.5\) (mumps) at

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\(^{15}\)A vaccine effectiveness rate of .8 is in the middle of the 70%-90% effectiveness range reported by Palache (1997, p. 844) for influenza vaccination among adults less than age 60. For mumps, controlled clinical trials yield an estimate of effectiveness of 95-96% for the mumps vaccine used in the United States, but eight outbreak studies yield estimates that range from to 75-91%, averaging 80% (Plotkin and Wharton, 1999, Table 13-6, p. 278).

\(^{16}\)The contact number for mumps in the winter months is 1.7 to 2.0 times higher than in the summer months (London and Yorke, 1973).
This section draws upon Boulier’s (2006) analysis of the demand for vaccinations. The assumptions that all vaccinations are purchased before the onset of the epidemic is standard (cf., Brito et al. 1991; Francis, 1997). Geoffard and Philipson (1997) assume vaccination occurs at birth. Francis (2004) is the only paper that examines the time pattern of vaccination during an epidemic. He shows that rational individuals with perfect foresight will stop obtaining vaccinations earlier than would be optimal, leading to an inefficient level of vaccination.

4. The Marginal Private Benefits of Vaccination

We assume individuals purchase vaccinations based on their estimates of the marginal private benefit of a vaccination. The difference between marginal social benefit and marginal private benefit is the externality of a vaccination. In this section we use a simple model of demand for vaccinations to examine demand determinants and thus the marginal private benefits to allow analysis of externalities in Section 5.17

In deriving the demand for vaccinations, we assume that individuals are identical and risk neutral, that all vaccinations are purchased before the onset of the epidemic, and that each individual incurs a cost \( k \) if infected.\(^{18}\) A susceptible individual chooses to purchase a vaccination if the expected marginal private benefit is greater than the price \( P \) of a vaccination. For an unvaccinated individual, the probability of becoming infected given that \( V \) people are vaccinated is \( p(V) \). If the cost of the disease is \( k \), then the expected cost of disease is \( p(V)k \). If the individual is inoculated with a vaccine of effectiveness \( m \), the expected cost is \( (1-m)p(V)k \). The marginal private benefit of a vaccination is the difference between the two: \( mp(V)k \). \( mp(V) \) is the expected number of illnesses avoided by obtaining a vaccination. Since the probability that an unvaccinated individual becomes ill is the same as the probability that a vaccinated individual whose inoculation is ineffective becomes ill, \( p(V) \) can be estimated as:

\[
p(V) = \frac{[R(\infty, V) - I_o]}{N - mV - I_o}, \tag{15}
\]

where \( R(\infty, V) \) is the number of “ever-infected” given \( V \) vaccinations. The numerator of (15) is the number of cases of the disease among the initially susceptible population and the denominator is the size of the susceptible population.

Dividing the numerator and denominator by \( N \), we can express (15) in terms of proportions of the population:

\[
p(V) = \frac{[r - i_o]}{1 - mv - i_o}, \tag{16}
\]

where \( v \) (=\( V/N \)) is the proportion of the population that is vaccinated.

Following Philipson (1996), Geoffard and Philipson (1997), and Francis (2004), we assume that individuals are forward looking and can estimate both the risk

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\(^{17}\)This section draws upon Boulier’s (2006) analysis of the demand for vaccinations.

\(^{18}\)The assumptions that all vaccinations are purchased before the onset of the epidemic is standard (cf., Brito et al. 1991; Francis, 1997). Geoffard and Philipson (1997) assume vaccination occurs at birth. Francis (2004) is the only paper that examines the time pattern of vaccination during an epidemic. He shows that rational individuals with perfect foresight will stop obtaining vaccinations earlier than would be optimal, leading to an inefficient level of vaccination.
of catching the disease at each level of vaccination and the number of vaccinations that would be purchased at each price. At any price \( P \) there is an equilibrium number of vaccinations \((V*)\) at which the expected number of vaccinations obtained from \( P = mp(V*)k \) equals \( V* \). Thus, \( mp(V*)k \) traces out the demand (or marginal private benefit) curve.

The following are some propositions about the demand curve obtained from the SIR model:

1. **The intercept of the demand curve must be less than \( k \), since not all individuals catch the disease even when there are no vaccinations.** That is, when \( V = 0 \), \( p(V) < 1 \). See Figure 2 which assumes \( k = 1 \).

2. **The higher is the cost of illness (\( k \)), the greater is the demand for vaccinations.** Willingness to pay rises with the cost of the disease.

3. **The demand curve is downward sloping, since \( p(V) \) declines with an increase in the number of vaccinations (i.e., a decline in the susceptible population).** For influenza with \( m = 1 \) (demand curve D1 in Figure 2), \( p(V) = .51 \) when \( V = 1 \). Thus, if the price of vaccinations were as high as .51, only one vaccination would be purchased. The demand curve drops sharply as the number of vaccinations increases, since changes in vaccination levels have a relatively large effect on the probability of infection. In fact, \( p(V) \) declines to .25 when less than 20% of the initially susceptible population is infected \((V = 182)\) and to less than .04 when about 30% of the population is vaccinated \((V = 297)\).

4. **The more infectious the disease, the greater is the demand for vaccinations.** Holding \( N \) and \( m \) constant, \( p(V) \) increases with the infectiousness of the disease. Compare the demand curves D1 for influenza and D3 for mumps for \( m \)

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19 Consistent with this assumption, Philipson (1996) finds that parents vaccinate their children at earlier ages for measles in states in which the past prevalence of measles is higher, and Mullahy (1999) finds that individuals’ demands for influenza vaccinations in the United States in 1991 are positively related to the severity of outbreaks of influenza in the previous year in the states in which they live.

20 Although his formulation of the problem differs somewhat from ours, Francis’s procedure for calculating the number of individuals who would purchase vaccinations at each price (Francis, 2004, pp. 2044-45) is essentially identical to the one we have used.

21 The demand curve is concave to the origin \((p''(V) < 0)\) in the epidemic case when the number of vaccinations is less than or equal to the number of vaccinations that would maximize the marginal effect of a vaccination. The demand curve is convex \((p''(V) > 0)\) if the number of vaccinations is sufficient to reduce the susceptible population to the non-epidemic case. Thus, the demand curve switches from concave to convex in the range \(1/\sigma + (r(\sigma) - i_0)/2 > s_0 \geq 1/\sigma\).
Figure 2. Demand Curves for Influenza and Mumps Vaccines of Differing Efficacy, $k = 1$ and $N = 1000$
(5) The demand for vaccinations does not vary monotonically with the effectiveness of the vaccine (m). Figure 2 compares demand curves for vaccines of 100% effectiveness and 80% effectiveness for influenza (D1 vs. D2) and mumps (D3 vs. D4). At low levels of vaccination, the demand for the 100% effective vaccine exceeds that of the less effective vaccine, since the risks of getting the disease for the unvaccinated \( p(V) \) are similar but the marginal benefit of the vaccination is higher for a vaccine that is 100% effective. However, at high levels of vaccination, the risk of getting the disease among the unvaccinated is sufficiently higher with the less effective vaccine (since there are many vaccinated individuals who are still potential sources of infection) that it is still worthwhile paying a high price for the vaccine even though it is less effective.

5. Marginal Externalities of Vaccination

In this section, we describe the pattern of marginal externalities from vaccination and show how this pattern is affected by the number of vaccinations, the infectiousness of the disease, and vaccine efficacy. As in the previous section, we assume that individuals are identical and risk neutral, that all vaccinations are purchased before the onset of the epidemic, and that each individual incurs a cost \( k \) if infected. With these assumptions, the marginal social benefit of a vaccination is simply \( k \) times the marginal effect of a vaccination (i.e., the number of illnesses prevented by an additional vaccination). Marginal private benefits of vaccination were described in the previous section. The marginal externality is the difference between marginal social and private benefits. We start by describing externalities for mumps and influenza and show how they vary with the number of vaccinations. We then examine how marginal externalities are affected by the infectiousness of a disease.

Figures 3 and 4 show the marginal social benefit and marginal private benefit (demand) curves for mumps and influenza vaccine, respectively, assuming that vaccination is 100% effective and \( k = 1 \). When \( k = 1 \), marginal social benefit can be interpreted as the change in illnesses resulting from an additional vaccination and marginal private benefit represents the expected illness avoided by the marginal vaccinee. The difference between the two curves is the marginal externality of a vaccination, depicted explicitly as curves E1 and E3 in Figure 5. Note that the pattern of rising and then falling marginal externalities displayed in our diagrams are quite different from those typically shown in textbooks. For example, Folland et al. (1993, p. 437) show the divergence between marginal social benefit and private demand curves as initially being large at low levels of vaccination and narrowing as the number of vaccinations increases. Phelps (1997, p. 511), the Office of Health Economics’ interactive internet text on The Economics of Health Care (2002, p. 30), and Santerre and Neun (2004, p. 233) show the marginal social benefit curve as upward parallel shifts of the private demand curve, implicitly assuming that the marginal externality is constant as the number of vaccinations increases. Henderson (2005, p. 75) portrays

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22It should be pointed out that these demand curves for vaccinations implicitly assume that the costs \( (k) \) of upon catching the disease are identical for the two diseases.
Figure 3. Marginal Private and Social Benefits of Mumps Vaccination, m = 1, k = 1, N = 1000, Io = 1
Figure 4. Marginal Private and Social Benefits of Influenza Vaccination, $m = 1$, $k = 1$, $N = 1000$, $I_0 = 1$
Figure 5. Marginal Externalities of Vaccination for Influenza and Mumps for Vaccines of Differing Efficacy, \( k = 1, N = 1000, I_o = 1 \)
the marginal externality as rising with the number of vaccinations.

The marginal externalities associated with vaccination in these examples illustrate the following propositions:

**Proposition 1. If the fraction of the population that is susceptible is sufficiently large and the disease is sufficiently infectious, the marginal externality of vaccination can be large, exceeding more than one case of disease prevented among the unvaccinated.**

Measuring marginal social and private benefit as the number of illnesses prevented by an additional vaccination, we have shown in the epidemic case that the marginal social benefit of a vaccination must be greater than one (if \( m = 1 \)) and the marginal private benefit must be less than one. The difference (the marginal externality) can be quite large as shown in Figure 5. With \( m = 1 \), the marginal externality for influenza vaccinations is greater than 1 for the first 283 vaccinations and peaks at 1.58 for the 223rd vaccination. For mumps, the marginal externality exceeds 1 in the range of 736-815 vaccinations and peaks at 1.38 for 794 vaccinations.

Because the maximum marginal effect of a vaccination in this version of the SIR model is less than 2 with 100% effective vaccination (see the discussion of equation 14), the maximum marginal externality of an additional vaccination must also be less than 2 cases of disease prevented among unvaccinated susceptibles.

**Proposition 2. The marginal externality of vaccination is not constant and may either rise or fall as the number of vaccinations increases.**

For mumps with \( m = 1 \), the marginal externality of a vaccination (Figure 3 and E3 in Figure 5) is initially small, rises to reach a peak as the number of vaccinations increases, and then declines sharply, approaching zero as nearly all individuals are vaccinated. This shape of the marginal externality curve has the following intuitive explanation. When the disease is sufficiently infectious, the susceptible population is sufficiently large and no one is vaccinated, the probability that an unvaccinated individual will catch the disease is large. Consequently, the marginal private benefit of the first vaccination is high, although less than one. However, the marginal social benefit of the first vaccination, which we have shown must exceed one in the epidemic case, may also not be much higher than one for a sufficiently large susceptible population. Removing one individual through vaccination from a large susceptible population still leaves many individuals who can become infected and transmit the disease to others. As a result, the marginal externality (i.e., the number of cases prevented among the unvaccinated from the first vaccination) will be small. For mumps, with \( m = 1 \), the marginal social benefit of the first vaccination is 1.019 and the marginal private benefit is .996, so that the marginal externality of the first vaccination is only .023.

If the initial fraction of the population that is susceptible is larger than that which would maximize the marginal social benefit of a vaccination, the marginal social benefit of a vaccination increases with a rise in vaccinations. However, the marginal private benefit of a vaccination declines with additional vaccinations since the probability that any unvaccinated individual will catch the disease falls. Consequently,
the difference between these two curves (the marginal externality) must rise initially. Eventually, however, the marginal externality must decline as the number of vaccinations increases. Consider the case in which almost all individuals are vaccinated. In this case, the marginal social benefit and marginal private benefit of vaccination are both small and the difference between them (the marginal externality) is also small. In fact, if all but one susceptible person is vaccinated and vaccine efficacy is 100%, the marginal social and private benefits of vaccinating that remaining susceptible individual are identical, since there are no unvaccinated people other than that individual who can benefit from the last vaccination. Consequently, the marginal externality from vaccinating that sole remaining susceptible person is zero.\(^{23}\)

The marginal externalities for influenza also rise and then fall as the number of vaccinations increases (Figure 4 and E1 in Figure 5). However, because influenza is less infectious than mumps, the marginal social benefit of the first vaccination is substantial (1.61) and the marginal private benefit is low (.51), so that the externality from the first vaccination is relatively large (1.10).

Marginal externalities do not always rise as the number of vaccinations increases. Suppose, for example, that 222 of the 1,000 individuals were immune from influenza because they had previously contracted the same strain in a previous epidemic. In this case, the marginal externality of the first vaccination (1.58) would equal the marginal externality of the 223rd vaccination when all individuals were susceptible. Subsequent vaccinations would have smaller externalities.

Proposition 3. The magnitude of the marginal externality of vaccination is not monotonically related to the infectiousness of the disease. That is, the marginal externality of a vaccination may rise or fall as infectiousness increases.

The marginal externalities of vaccination for influenza and mumps for \( m = 1 \) are shown in Figures 3 and 4 and also by curves E1 and E3 in Figure 5. Below 351 vaccinations the marginal externalities for influenza exceed those of mumps; for 351 or more vaccinations the marginal externalities for influenza are smaller than for mumps. At low levels of vaccination, the marginal social benefit of an influenza vaccination is large and the marginal private benefit is low, so that the marginal externality is large. For mumps, on the other hand, the marginal social benefit is low and the marginal private benefit is high, so that the marginal externality is small. When the number of vaccinations increases, the marginal social benefit of influenza vaccination rises to a peak and then drops sharply, resulting in a substantial fall in the externality. However, at these higher levels of vaccination for mumps, eliminating a potential infective does have an appreciable effect on the number of cases of mumps, so that the marginal social benefit and marginal externality are both large.

At any given fraction of the population susceptible, the marginal externality is zero when the disease is not at all infectious, rises to reach a maximum as \( \sigma \) increases, and then falls for additional increases in \( \sigma \). For example, when half the population of 1000 is susceptible with 1 initial infective and a perfectly effective vaccine, the marginal externality is less than one if \( \sigma < 2.02 \). The marginal externality

\(^{23}\) If \( m < 1 \), then the externality of the last person vaccinated is still positive, since inoculated individuals for whom the vaccine was ineffective are still susceptible.
peaks at 1.52 when $\sigma = 2.21$ and then decreases with further increases in infectiousness.

**Proposition 4.** The magnitude of the marginal externality of vaccination is not monotonically related to the effectiveness of the vaccine.

Figure 5 shows the marginal externality of vaccination for differing levels of vaccine efficacy ($m$) for influenza and mumps. When the level of vaccinations is low, the marginal externality of the more effective vaccine is higher. Compare curves E1 ($m = 1$) and E2 ($m = .8$) for influenza. However, as the number of vaccinations increases, both the marginal social benefit and marginal private benefit of vaccination change non-linearly. At some point the marginal social benefit and the marginal externality of the more effective vaccine declines rapidly as the susceptible fraction is diminished below the epidemic level. However, at the same number of less effective vaccinations, the susceptible population is sufficiently great to sustain an epidemic, so that the marginal social benefit and marginal externality of the less effective vaccine remain large. For example, in Figure 5 for influenza the marginal externality of vaccination when $m = 1.0$ exceeds the marginal externality when $m = .8$ up to 271 vaccinations. Thereafter, the marginal externality of the less effective vaccine is larger.

### 6. Conclusions

There has been little analytical or empirical work trying to quantify the magnitude of the marginal social benefits and externalities of vaccination. This paper helps to fill that gap by adapting the Susceptible-Infectious-Removed (SIR) epidemic model to investigate how the magnitude of marginal social benefits and externalities associated with vaccination varies with the number of vaccinations, the efficacy of the vaccine, and the infectiousness of the disease. We illustrate the pattern and sizes of the marginal social benefits and externalities associated with influenza and mumps vaccinations.

Five positive and negative findings are of particular interest. The most striking positive finding is that the actual size of the vaccination externality can be large at some levels of vaccination. In particular, for some of our influenza simulations, the marginal externality can exceed one case of disease prevented among the nonvaccinated for each additional vaccination. A second striking positive finding is that the marginal externality of vaccination may rise and then fall with increases in the fraction of the population vaccinated. The exact pattern depends on the infectiousness of the disease, and the effectiveness of the vaccine. Third, the patterns of externalities we find are quite different from, and more complex than, the diagrammatic presentations found in standard microeconomics or health economics textbooks. Fourth, the marginal social benefit of vaccination need not be monotonically related to the infectiousness of a disease. Fifth, externalities need not vary monotonically with vaccine efficacy, or the infectiousness of the disease.

The remainder of this conclusion considers (i) whether some limitations of the SIR model affect the applicability of our results, and (ii) additional implications of our findings for the existing literature.
The SIR modeling approach we adopted embodies some important simplifications:

1. All susceptible individuals are equally likely to become infected upon coming in contact with an infected person.
2. The contact rate of susceptibles and infectives is constant, each contact is with a random sample of the population in each period, and the contact and transmission rates are exogenous to the risk of getting the disease.
3. Individuals are identical, risk neutral and incur a fixed cost if infected.
4. Population size is fixed and there are no births, deaths, or migration.

We think that future research efforts to relax these simplifications would be worthwhile, although they are likely to reinforce our finding that the pattern of marginal social benefits and externalities of vaccination will depend in important ways on the characteristics of a disease outbreak. For example, Philipson (1995, pp. 218-219) has sharply criticized the set of assumptions given in item 2 in this list, arguing that contact rates and transmission rates vary with disease prevalence. Suppose, in the case of influenza, that an increase in the fraction vaccinated reduces the probability that unvaccinated individuals will become infected and in turn decreases the frequency with which they wash their hands and increases the frequency of attending crowded movie theaters. A consequence would be that the transmission parameter $\beta$ and thus the contact number $\sigma$ would rise with an increase in the fraction vaccinated. Since, however, there is a non-monotonic relationship of marginal social benefits with increases in infectiousness, it is not readily apparent how the marginal social benefits and externalities of vaccination would change. While one could simulate these changes, we are unaware of any empirical evidence for influenza that shows how changes in prevalence lead to changes in preventive behaviors and in turn how these behaviors affect the transmission parameter.

Our modeling also yields some additional implications about the current literature. First, because externalities associated with vaccination can be large, ignoring these external benefits in cost-benefit or cost-effectiveness analyses may understate the true social benefits of vaccination. The modeling approach we have described suggests how one might estimate the magnitudes of these externalities.

In his examination of optimal production and consumption of mumps vaccine, Boulier (2006) provides some instructive numbers. If mumps vaccine were perfectly effective, the present value of the costs of mumps ($k$) less the expected value of side effects per vaccination would equal $300 (in 2002 U.S. dollars). The cost ($k$) includes: (1) the direct expenditures for physician and hospital visits and the present value of treating hearing impairment, (2) the indirect value of lost time/earnings for uncomplicated and complicated cases, and (3) the present value of lost earnings due to death and disability. The estimated cost of a vaccination is $63, which includes the marginal cost of producing the vaccine plus the price of a doctor’s visit and the marginal value of time for the individual purchasing the vaccination. If vaccine were perfectly competitively supplied, the number of vaccinations would be about 5% below the social optimum and a subsidy of $44 would need to be paid to producers (or consumers) to achieve the social optimum. With a vaccine that is 80% effective (as is Mumpsvax, the vaccine used in the United States), universal vaccination is optimal.
A second implication is that incorporating externalities into conventional cost-benefit or cost-effectiveness analysis is likely to be difficult because the magnitudes of externalities vary with the overall rate of vaccination. For example, the World Bank has assumed that externalities are a constant multiple of infant vaccinations to assess benefits of immunization programs (Cowley et al. 1993). Our analysis suggests that the appropriate multiplier will depend upon the infectiousness of the disease, levels of treatment or coverage of a program, and vaccine efficacy.

Third, statistical studies of the effectiveness of vaccination in preventing illness that do not take externalities into account may yield biased estimates. Consider Bridges et al. (2000) study of the effectiveness of influenza vaccination in preventing respiratory illness among a sample of workers at a Ford Motor Company plant in Dearborn, Michigan. One group of workers was given influenza vaccinations and a second group was given a placebo. The authors measured rates of illness between the two groups and then estimated the benefits of vaccination by comparing the differences in costs per worker of lost workdays and medical care between the two groups. From this comparison, they drew the conclusion that vaccinating healthy young adults was not worthwhile since the estimated benefits per worker were less than the cost of vaccination. The problem with this comparison is that externalities from the vaccinated workers may have reduced the incidence of illness among unvaccinated workers. Consequently the measured differences in illness rates and costs of lost work days and medical care between the two groups would underestimate the effectiveness of the vaccine in preventing disease and the benefits of vaccination. (See Haber, 1999 for an analysis of biases in conventional measures of effectiveness in vaccine trials.)

Finally, introducing more complexity into the model to account for non-random mixing of populations and differential susceptibilities to disease could provide better evaluation of strategies for designing vaccination programs. As noted in the introduction, the most recent influenza vaccination recommendations of the Advisory Committee on Immunization Practices (N. Smith et al. 2006, p. 2) concentrate on persons at high risk for influenza-related complications or death. The CDC studies of the economic impact of pandemic influenza in the United States by Meltzer et al. (1999a, 1999b) similarly focus on identifying groups at high risk of contracting illness and the associated mortality and economic costs. These studies suggest that the elderly should receive highest priority for vaccination if risk of death were the criterion, but the lowest priority if net economic returns to vaccination are considered. Individuals ages 0-19 and 20-64 with preexisting medical conditions that leave them at high risk for illness requiring treatment and hospitalization should receive the highest priority when net economic returns are considered. The recommendations by the Advisory Committee and the analyses by Meltzer et al. ignore externalities.

Consideration of the transmission of disease by sub-groups of the population could alter priorities. For example, if school children are more likely to catch and transmit diseases than are (say) the elderly or the working-age population with preexisting conditions, then optimal design of a program with limited resources might well dictate a different pattern of vaccination. Ackerman et al. (1984, pp. 144-146) argue that “(t)he experience with Asian-flu epidemics suggested that vaccination of school children would provide an effective block against spread through the community. Not only did schools have high attack rates, but they provided the paths of spread between families and neighborhoods that would not usually exist if the
schools were closed.” (p. 144). Halloran et al. (2001) build a stochastic simulation model that explicitly takes into account population heterogeneity with respect to contact rates and susceptibility to disease and finds that high levels of vaccination of children (e.g., 70% coverage) have dramatic effects on the rates of adult influenza. Reichert et al. (2001) provide some direct evidence on this point. They find that excess mortality from influenza and pneumonia among the elderly rose when Japan relaxed its requirement that school children be vaccinated in 1986 and dropped the requirement altogether in 1994.

The work by Halloran et al. (2001) noted above is one of a series of papers by Halloran, Longini and their coauthors that use improvements in modeling methods, better data, and increased computational power to examine vaccination strategies. Longini et al. (2005) estimates magnitudes of vaccination, antiviral prophylaxis, or quarantine measures in containing the spread of pandemic influenza in a simulated outbreak in Southeast Asia. Patel et al. (2005) calculate how the allocation of a fixed stock of vaccine across age groups would affect the magnitude of illness and mortality that might result from a pandemic flu outbreak. These simulations incorporate both the direct and indirect effects of vaccination. The kinds of simulations performed in these models might provide very useful information on benefit-cost analysis of vaccination strategies and identification of optimal strategies if these models were married with economic analysis on the costs of illness and mortality and the expenditures needed to implement the vaccination strategies.

References


